Flexible Phased Array Shape Reconstruction

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Abstract — Lightweight, flexible phased arrays enable new applications by dynamically changing shape during deployment and operation. However, these shape changes must be tracked and accounted for by changing the element excitation in order for the array to continue proper operation. We propose a framework for reconstructing the shape of a flexible phased array using only measurements of mutual coupling between the antennas in the array. The framework is demonstrated using a passive 2.5 GHz phased array and fixed to 8 different frames of known radii of convex and concave curvature. Our results show the ability to reconstruct shape to within $\approx 0.04\lambda$ position error, even in cases where the phased array is bent dramatically, without any advanced knowledge or additional information. The framework is modular and can be easily adapted to other phased array systems with different antennas, frequencies, and physical constraints.

Keywords — Signal Processing, Flexible Electronics, Phased Array, Shape Sensing, EDM

I. INTRODUCTION

The recent emergence of flexible integrated circuit based arrays [1] [2] has created a need for in-situ determination of array shape. Radio frequency integrated circuits enable lightweight, flexible arrays by combining the functions performed by bulky and heavy discrete components into a small, low mass, and low profile package. Unlike a traditional rigid, conformal array such as [3], the shape of a flexible phased array can change continuously as it operates. A depiction of a hypothetical flexible phased array is shown in Fig. 1. While these lightweight and flexible arrays enable new applications, the movement of the phase centers of the array elements undermines the array’s ability to perform beam-steering, null-creation, wavefront-engineering, and beam-focusing - unless it is accounted for. Array shape reconstruction, the task of determining where the elements of an array are located, is critical for unlocking the full potential of flexible integrated circuit arrays. Specialized shape sensing hardware can be added [4], but this additional complexity and cost can be avoided by using the microwave transceivers already present in a phased array as sensors. [5] demonstrated that changes in relative array element position induced changes in coupling measurements but did not demonstrate shape reconstruction. In this work, we propose a framework for achieving full shape reconstruction using only mutual coupling measurements within the array. This framework is used to accurately reconstruct the shape of a connectorized 1D 8-element flexible array bent to concave and convex shapes.

II. SHAPE RECONSTRUCTION - SPIRAL MATCH

The proposed shape reconstruction process is depicted in Fig. 2. At a high level, the shape reconstruction algorithm takes in mutual coupling measurements between elements in an array and outputs the relative positions of the elements. We take a two step approach, first mapping individual mutual coupling measurements to a physical constraint (in this case: distance between the phase centers of an antenna pair), then using an optimization algorithm to turn those physical constraints into array shape. In this work, we designed an algorithm called "Spiral Match" to address the first mapping and used semi-definite relaxation to address the second. The result is the reconstructed shape of a flexible array.

For a reconstruction algorithm to be used, a bijection between mutual coupling and distance must be generated. Mutual coupling in phased arrays is multi-faceted phenomenon as reactive near-fields, occlusion (blocked line of sight), and multi-path reflections, and other effects contribute to the observed coupling between elements. However, we propose a simplified model, rooted in the underlying physics, with sufficient accuracy for shape reconstruction and adaptability for use with a variety of array types.

Spiral Match is a transformation which projects a coupling measurement onto a model for coupling as a function of...
distance and uses this to infer the distance. We begin with a simplified far-field model for RF coupling:

\[ A_{ij} = \frac{a_i D_i(\theta_{ij})a_j D_j(\theta_{ji})}{|\vec{l}_{ij}|^2} e^{-\imath(k \phi_i + \phi_j + k|\vec{l}_{ij}|)} \]

(1)

\[ \theta_{ij} = \cos^{-1} \left( \frac{\vec{r}_i \cdot \vec{r}_j}{|\vec{l}_{ij}|} \right) \]

(2)

where \(a_i\) is the constant gain offset for antenna \(i\), \(D_i(\theta)\) gives the directivity of antenna \(i\) for an angle \(\theta\) relative to broadside, \(\vec{r}_{ij}\) is the vector pointing from the phase center of antenna \(i\) to the phase center of antenna \(j\), \(\phi_i\) is constant phase offset for antenna \(i\), and \(\vec{r}_i\) is the antenna position vector, normal to the ground plane.

Using far-field transmission assumptions, the phase is proportional to the distance between antennas \(|\vec{l}_{ij}|\) and the coupling power falls off as \(1/|\vec{l}_{ij}|^2\). Decreasing amplitude and linear phase progression, mapped in polar coordinates, draws a spiral as a function of distance, hence the name "Spiral Match". While these trends approximately hold as a flexible array experiences relative rotation during bending. To account for the change in coupling due to this rotation we incorporate the far field radiation pattern of the element antennas. We used an FDTD simulation to obtain this pattern, though it can also be determined through analysis or measurement.

Next, we eliminate the constant phase and amplitude offset terms with a baseline measurement of the array in the flat configuration. We take the quotient \(H = \frac{1}{\lambda_{meas}}\) to generate a transfer function which is a function of the bent distance:

\[ H_{ij}(|\vec{l}_{ij}|) = \frac{D(\theta_{ij})D(\theta_{ji})}{D\left(\frac{\pi}{2}\right)^2} \frac{|\vec{l}_{ij}|_{flat}^2}{|\vec{l}_{ij}|^2} e^{-\imath(k(|\vec{l}_{ij}|_{flat} - |\vec{l}_{ij}|))} \]

(3)

where \(\theta_{ij} = \frac{\pi}{2}\) and \(|\vec{l}_{ij}|_{flat}\) (antenna pitch for a flat array) are known constants. \(\theta_{ij}\) is a function of \(|\vec{l}_{ij}|\), a bijection made possible through the assumption of the shape having constant curvature. This is our final model for mutual coupling as a function of distance. To solve for the Euclidean distance, we define the following optimization, which corresponds to a projection onto a spiral in polar coordinates, (depicted visually in Fig. 2):

\[ E_{ij} = \arg \min_d ||H_{ij}(d) - H_{ij}^{meas}|| \]

(4)

We implement our model iteratively, adding additional constraints after each iteration. The first pass does not include the directivity terms and is used to determine shape polarity (convex or concave). The second pass incorporates the directivity, assuming constant curvature in order to determine a value for \(\theta_{ij}\), and constrains the result to physically realistic distances (an example of this simple physical constraint is that elements connected by 1.2\(\lambda\) of flexible ground plane cannot be bent such that their separation is 2\(\lambda\)). It is worth noting that the assumption of constant curvature can introduce error but only exists in the determination of \(\theta_{ij}\) for a single element pair and this assumption does not preclude the possibility of a reconstructed shape with non-constant curvature. The result of this iteration is the output of the Spiral Match algorithm.

### III. Shape Reconstruction - Semi-definite Relaxation

The result of mapping the mutual coupling measurements to a matrix of pairwise distances is a Euclidean distance matrix (EDM). EDMs are well-studied matrices with a variety of applications in disparate disciplines [6]. Reconstructing relative position from a perfect (noise-free, correctly-labeled, dense) EDM is a trivial eigenvalue problem that is referred to as multi-dimensional scaling. However, doing so on a noisy, incorrectly-labeled, or sparse EDM will often yield imaginary eigenvalues, implying that our EDM does not correspond to any physically possible point cloud in the desired dimension. To get around this restriction, a number of algorithms have been developed which correct an erroneous EDM to a consistent and physically realistic EDM which guarantees real eigenvalues. Each algorithm caters to different physical systems with different deficiencies. We chose semi-definite relaxation (SDR) for its strong performance using EDMs with deletions or errors and its ability to incorporate a masking matrix, which tunes each entry’s contribution to the loss function. This tuning is used to suppress elements with low signal to noise ratio to avoid large errors. SDR is implemented as a convex search over a high-dimensional space, followed by an eigenvalue decomposition to obtain position.

### IV. Demonstration and Results

#### A. Measurement Set-up

We designed a 1D, 8-element passive array to demonstrate the shape reconstruction algorithm. The array consists of folded dipoles mounted with a pitch of 0.6\(\lambda\) at 2.5 GHz on a large, flexible copper ground plane. The array and the element input matching of the folded dipoles is shown in Fig. 3. In order to create known array shapes for measurement, the array was mounted on fixed-curvature, wooden frames with radii of curvature ranging from concave (\(R = 206\)mm) to flat (\(R \rightarrow \infty\)) to convex (\(R = -206\)mm). Here, the sign corresponds to the polarity of the bend (+ for concave, − for convex).

Photographs of the array mounted on the non-flat frames and the frame bend radius is shown in Fig. 4. The coupling between every pair of elements was measured for each of the shapes using a VNA inside a partial RF-anechoic chamber. These measurements amount to 9 matrices, \(A \in \mathbb{C}^{8 \times 8}\), on which the shape reconstruction algorithm is used.

#### B. Results

The accuracy of each shape reconstruction algorithm is quantified with two numbers. First, Spiral Match’s effectiveness is measured by the mean distance error for each element pair, \(\Delta E\). Second, the final shape reconstruction accuracy is measured by the mean error for each element position in the reconstruction \(\Delta x\). Both these errors are calculated by comparing the algorithm results with the known
Fig. 3. Connectorized 1D 8-element flexible phased array. Measured and simulated adjacent element S-parameters are also shown.

physical design dimensions of the frames on which the arrays are mounted. The error quantities ($\Delta E$ and $\Delta x$) and a visualization of the reconstructed shapes and the true shapes are given in Fig. 4.

V. CONCLUSION

In this work we demonstrated a framework for flexible phased array shape reconstruction using only measurements of local coupling, eschewing the need for external receivers or extraneous sensors. While this demonstration uses passive dipole phased array operating at 2.5 GHz, the method can easily accommodate other antenna candidates, frequencies, and physical constraints. The shape reconstruction framework predicts the Euclidean distance of all pairs of antennas in an $8 \times 1$ phased array to within $\approx 0.04 \lambda$. SDR of the resulting distance matrices reconstructs the shapes given noisy data without amplifying the error. This self-contained shape reconstruction framework which requires no additional sensors enables autonomous operation of lightweight flexible phased arrays in a range of environments and applications.

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REFERENCES


Fig. 4. Shape reconstruction results. Left column: Flexible array mounted on frames of known shape with curvature radius on top. Center column: Heatmap of EDM error with average on top. Right column: Reconstructed shape (red) overlayed on known shape (black) with average position error on top.