Performance Limits of Sub-Shot-Noise-Limited Balanced Detectors

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Abstract: We present an analysis of sub-shot-noise-limited coherent receivers for detecting quantum states of light. We introduce a noise model for coherent receivers and outline a guide to coherent receiver design for silicon photonics platforms.

1. Introduction

Quantum-noise-limited systems are crucial for high-sensitivity metrology, communications, and quantum-enhanced applications. Shot-noise-limited receivers, in particular, enable quantum state tomography and quantum light detection at room temperature with the possibility of large-scale integration with silicon photonics [1]. While there has been some work outlining the electronic design principles for shot-noise-limited receivers [2], there hasn't been a thorough guide for coherent receiver electronic-photonic co-design, taking common mode rejection ratio (CMRR), PD responsivity, electrical noise, and optical losses into account. In this work, we introduce a semi-classical noise model of balanced detection and discuss the parasitic effects of receiver design imperfections to guide the design of sub-shot-noise-limited receivers for large-scale quantum photonic integrated circuits.

2. Noise Model for Balanced Detection

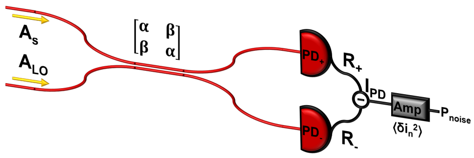

Fig. 1: Coherent receiver model.

We start with a semi-classical model of light. In this model, we quantize optical power with a focus only on intensity noise and link the shot noise to quantum fluctuations in light. The model is compatible with the classical electromagnetic fields in the classical picture and can be generalized to quantized electromagnetic fields in the quantum picture [3]. We define the optical power by a random variable P that can be split into signal-carrying and noise-carrying components, i.e. $P = p + \delta p$. $p = \langle P \rangle = \hbar\omega\langle n \rangle$ is the mean optical power corresponding to a mean photon number, $\langle n \rangle$, and $\delta p = P - \langle P \rangle$ is the random variable corresponding to shot noise. Then, for classical Poissonian light with large $\langle n \rangle$, $\langle \delta p^2 \rangle = (\hbar\omega\sqrt{\langle n \rangle})^2 = \hbar\omega p$, where $\hbar\omega$ is the photon energy per bandwidth.

Now, for a coherent receiver shown in Fig. 1, we derive the output current from the PDs. The incident complex electromagnetic fields on the signal and local oscillator (LO) ports are A_s and A_{LO}, respectively corresponding to $Re\{A_s\}^2 = P_s = p_s + \delta p_s$ and $Re\{A_{LO}\}^2 = P_{LO} = p_{LO} + \delta p_{LO}$. For an unbalanced case, we have a directional coupler with complex reflectivity and transmission coefficients of α and β, where $|\alpha|^2 + |\beta|^2 = 1$. Each PD outputs a current proportional to the optical power ($I = |E|^2 R$, where E is the incident electric field and R is the PD responsivity), and balanced PD pair output a current that is the difference between the currents on both branches. Imbalance in PDs can also be added by defining a different responsivity for each PD, R_+, and R_-. Then,

$$P_{signal} = \langle I_{PD} \rangle^2 = 4L^2 R_{sum}^2 \langle \sqrt{p_s + \delta p_s}\sqrt{p_{LO} + \delta p_{LO}} \rangle^2 \approx 4L^2 R_{sum}^2 p_s p_{LO} \tag{1}$$

$$\langle \Delta I_{PD}^2 \rangle \approx L^2 \left[R_{diff+}^2 \langle \delta p_s^2 \rangle + R_{diff-}^2 \langle \delta p_{LO}^2 \rangle + 2R_{diff+} R_{diff-} \langle \delta p_s \delta p_{LO} \rangle + 4R_{sum}^2 \left(\frac{p_s \langle \delta p_{LO}^2 \rangle}{4p_{LO}} + \frac{p_{LO} \langle \delta p_s^2 \rangle}{4p_s} + \frac{\langle \delta p_s \delta p_{LO} \rangle}{2} \right) \right] \tag{2}$$

where L is the lumped optical insertion loss of the system, $R_{diff\pm} = \pm R_\pm |\alpha|^2 \mp R_\mp |\beta|^2$, $R_{sum} = (R_+ + R_-)|\alpha||\beta|$. At high LO powers, only terms linear with p_{LO} dominate. We also consider electrical noise power ($\langle \delta i_n^2 \rangle$),

$$P_{noise} = \langle \Delta I_{PD}^2 \rangle + \langle \delta i_n^2 \rangle \approx L^2 \left[R_{diff-}^2 \langle \delta p_{LO}^2 \rangle + R_{sum}^2 \left(\frac{p_{LO} \langle \delta p_s^2 \rangle}{p_s} \right) \right] + \langle \delta i_n^2 \rangle \tag{3}$$

At the strong LO limit, this classical electrical noise becomes negligible, giving the SNR,

$$SNR = \frac{P_{signal}}{P_{noise}} = \frac{4p_s}{\left[\frac{R_{diff-}^2}{R_{sum}^2} \frac{\langle \delta p_{LO}^2 \rangle}{p_{LO}} + \frac{\langle \delta p_s^2 \rangle}{p_s} \right] + \frac{\langle \delta i_n^2 \rangle}{L^2 R_{sum}^2 p_{LO}}} \underset{p_{LO} \to \infty}{=} \frac{4p_s}{\frac{1}{\mathrm{CMRR}} \frac{\langle \delta p_{LO}^2 \rangle}{p_{LO}} + \frac{\langle \delta p_s^2 \rangle}{p_s}} \tag{4}$$

With this, as an example, we consider a squeezed signal with variance, $\langle \delta p_s^2 \rangle = s\hbar\omega p_s$, where $s \leq 1$ is the squeezing factor, and strong coherent LO with a classical relative intensity noise of RIN. Then, the noise power of a coherent receiver normalized to LO power is

$$\frac{P_{noise}}{p_{LO}} = L^2 \left[\frac{1 + \mathrm{RIN}^2 p_{LO}}{\mathrm{CMRR}} + s \right] \left[(R_+ + R_-)^2 |\alpha|^2 |\beta|^2 \right] \hbar\omega + \frac{\langle \delta i_n^2 \rangle}{p_{LO}} \quad (5)$$

3. Coherent Receiver Design

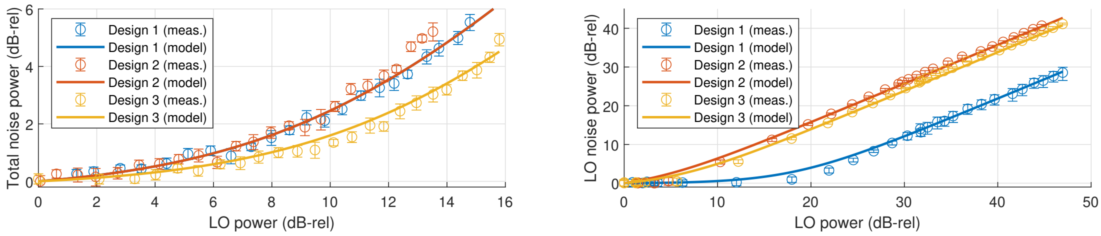

Fig. 2: a) Total and b) LO electrical noise power plots of receiver designs with varying optical LO power showing the distinction between signal and LO quantum-limited receivers. All designs have the same electrical noise.

When $s = 1$ and $\mathrm{RIN}^2 \ll \mathrm{CMRR}$ in (5), two key specifications of a quantum coherent receiver arise: Knee power from signal noise contribution (P_{knee}), the LO power for which the signal shot noise is equal to other noise, and shot noise clearance (SNC), the ratio of signal shot noise with maximum LO power (P_{max}) to other noise.

$$P_{knee} = \frac{\langle \delta i_n^2 \rangle}{L^2 R_{sum}^2 \hbar\omega \left(1 - \frac{1}{\mathrm{CMRR}}\right)} \quad (6) \qquad SNC = \frac{P_{max}}{P_{knee}} \quad (7)$$

When the LO noise term is lower than signal shot noise, $\frac{1+\mathrm{RIN}^2 p_{LO}}{\mathrm{CMRR}} < 1$, the receiver is *signal* shot noise limited, which is the regime quantum coherent receivers should operate in. When $\frac{1+\mathrm{RIN}^2 p_{LO}}{\mathrm{CMRR}} > 1$, the receiver is *LO* noise limited. Therefore, even for a quiet LO, LO shot noise can degrade the detection of quantum light, in the case of a squeezed signal, reducing the detected squeezing as much as $\frac{s}{\frac{1}{\mathrm{CMRR}} + s}$.

Thus, a receiver's quantum noise performance can be characterized by two measurements: total noise power with varying p_{LO} and LO/signal noise power (LO/signal contribution to the total noise power) with varying p_{LO}. The knee power from LO noise contribution is offset from the total knee power by the CMRR, while the total knee power is determined only by the electrical noise, optical loss, and responsivity. Since signal shot noise power, which determines how well a receiver can probe quantum light, is calculated by subtracting the LO noise power from the total noise power in the shot-noise-limited regime as seen in (5), signal noise performance is impacted similarly by these specifications as summarized in Fig. 3.

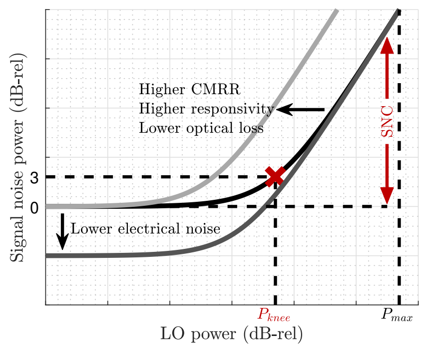

Fig. 3: Design guide to improve a coherent receiver's quantum noise performance.

To illustrate this, three designs (named designs 1, 2, and 3, respectively) were built: Balanced PDs ($R_+ = R_-$) with 50:50 directional coupler ($|\alpha|^2 = |\beta|^2 = 0.5$), balanced PDs with 90:10 directional coupler ($|\alpha|^2 = 1 - |\beta|^2 = 0.9$), and unbalanced PDs ($R_+ = 0$) with 50:50 directional coupler. Total shot noise power and LO shot noise power curves were characterized to verify the model. To compare LO shot noise contribution between designs, an amplitude modulation at 5.9 MHz was injected into LO, making it the dominant term in the numerator of the LO term in (5). As seen in Fig. 2, designs 1 and 2 have the same total knee powers, but different knee powers from LO, offset by the difference in CMRRs. Designs 1 and 3 have total knee powers offset by around 3 dB and knee powers offset by the difference in CMRRs due to different responsivities.

References

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